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By George W. Pierce.

(Continued from page 3 of cover.)

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1. Durand, Elias J.— The Genera Midotis, Ionomidotis and Cordierites. pp. 1–18. 2 pls. September, 1923. $.80.
2. Baxter, Gregory Paul and Scott, Arthur Ferdinand.— A Revision of the Atomic Weight of Boron. The Analysis of Boron Trichloride and Boron Tribromide. pp. 19–48. September, 1923. $.80.
3. Lipka, Joseph.— Trajectory Surfaces and a Generalization of the Principal Directions in any Space. pp. 49–77. September, 1923. $1.00.
4. Pierce, George W.— Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters. pp. 79–106. October, 1923. $1.00.

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Received August 13, 1923.

I. Introduction.

Professor W. G. Cady's paper [1] on the use of the mechanical vibration frequencies of piezoelectric crystals as standards of frequency of electric oscillations came to my attention while I was engaged in attempting a precision calibration of the wavemetric standards of Harvard University.

Professor Cady's suggestions proved to be of great value in this work, and it is thought that the interest and utility of the piezoelectric devices justifies the publication of this report.

Piezoelectric Resonators and Oscillators.— Cady's paper is devoted chiefly to the *piezoelectric resonator*, which gives a *transient* response when an electric oscillating circuit is made to pass through a period synchronous with the mechanical period of a parallelopiped cut from a piezoelectric crystal. He gives, however, a description also of the *piezoelectric oscillator*, which gives *sustained* oscillations. In Cady's apparatus this oscillator consists of a parallelopiped of quartz crystal with two pairs of electrodes,— one to be connected into the first grid circuit and the other into the last plate circuit of an amplifier train with resistance coupling between stages. With this device he obtained *sustained oscillations* with a period fixed by the mechanical period of the quartz alone.

In my experiments I have been able by a change of connections to use a plate of piezoelectric quartz connected to a single vacuum tube (with or without inductance or capacity in the circuit) to constitute a *piezoelectric oscillator* (described in II) capable of giving sustained electric oscillation with a constant frequency determined by the frequency of the crystal alone. By means of this single fixed frequency and its harmonics used in coöperation with a variable electric oscillator and its harmonics I have found it possible to calibrate completely and

[1] W. G. Cady: The Piezoelectric Resonator, Proc. Inst. Radio Engineers, Vol. 10, p. 83, 1922.

by fine steps a series of wavemeters extending in range from 50 meters (6000 kilocycles per second) to 50000 meters (6 kilocycles per second), and to obtain finally the absolute period of the crystal oscillator itself in terms of a standard clock by the intermediation of a tuning fork.

Having obtained this calibration with an accuracy of about one-tenth of one per cent over the entire range, a number of crystal resonators (made according to Cady's description) have been standardized as to fundamental and partial frequencies.

II. Improved Piezoelectric Oscillator.

The piezoelectric oscillator here employed has the circuit shown in Figure 1. The plate circuit of a vacuum tube contains posts, shunted by the bypass condenser C, of about .001 microfarad, to which a

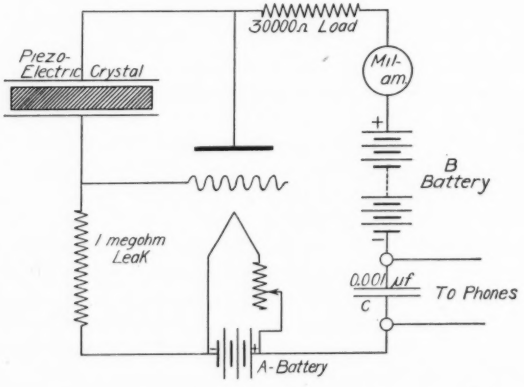

Figure 1. Diagram of Connections of Improved Piezoelectric Oscillator.

telephone or the *input* of an audio frequency amplifier may be connected, in series with a B-Battery and a resistance load of 30,000 Ω. The grid is connected to the filament by a leak resistance of about one megohm; and the piezoelectric crystal provided with electrodes is connected between the plate and grid.

In practice the 30,000 Ω plate resistance, which will be called the "Load," may be replaced by an inductance of any value if large enough (say greater than 10 milhenries for a crystal of period 420,000

cycles per second). This inductance may be used to couple the piezoelectric oscillator to any other circuit. Beats with another circuit are, however, often loud and clear with no coupling other than static effects, so that the resistance " load," or even a very large iron-core inductance may be used at the position marked " load."

Mounting for Quartz Crystal of Piezoelectric Oscillator.—A plate of quartz of thickness 0.5 cm., width 4.5 cm. and length 6.2 cm. on top surface and 7.0 cm. on bottom surface, was mounted, as in Figure 2, loosely within a hard rubber frame which was a little thicker

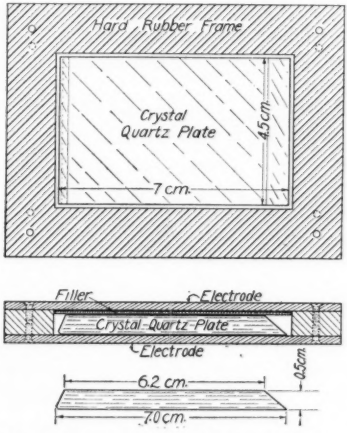

FIGURE 2. Mounting of Quartz Plate.

than the quartz. Sheets of brass to serve as electrodes were screwed to the two faces of the frame.

A thin sheet of spring brass (marked " filler ") was inserted between one of the electrodes and the quartz, in such a way that only a light pressure was exerted on the quartz. The particular piece of quartz shown in Figure 2 was supplied to me by Professor E. L. Chaffee from his optical collection, and was employed without modification. This accounts for its peculiarly beveled ends, which are for present purposes accidental.

The optic axis of the specimen was determined experimentally to be parallel to the 7-cm. edge. Other facts regarding the original orientation of the plate in the natural crystal have not been determined.

The specimen when tested as a piezoelectric *resonator* exhibits six or eight prominent resonant frequencies and many hundred resonant frequencies of weaker response. When connected between the plate and grid of the vacuum tube as in Fig. 1, so as to constitute an *oscillator*, the apparatus gives persistently sustained electric oscillations of frequency 419,640 cycles per second, as was found by methods described below. This frequency is probably the mechanical frequency of elastic vibration of compression in the direction of the shortest dimension of the plate, although this period is also conditioned somewhat by the length and breadth of the plate.

The persistent oscillatory current in the circuit is evidenced by a drop of the current in the plate milammeter when the crystal is connected, or by heterodyne effects when used in proximity to another oscillator.

Different tubes and plate voltages were employed. For wavemeter calibration a Radiotron UV 199 with a 44-volt plate battery was highly satisfactory. With a power-tube and a 500-volt plate battery several watts of oscillatory power could be obtained.

Constancy of Frequency.— The quartz oscillator was found not to change its frequency by as much as three one hundredths of one per cent for a change of temperature from 8°C to 24°C. Also a weight of 2 Kg. applied as a pressure to one of the electrodes did not change the frequency perceptibly. Variation of filament current, plate voltage, resistance, inductance, capacity, or any kind of electrical tuning of the piezoelectric oscillator had no effect as great as three one hundredths of one per cent on the frequency of the oscillation.

III. Precision Calibration of Wavemeter.

The general arrangement of apparatus for the calibration of a wavemeter in terms of the constant oscillation frequency of the piezoelectric oscillator is shown in Fig. 3. At WM is shown the wavemeter consisting of a General Radio Precision Condenser, Type 222, and having at L, interchangeable coils known in this report as coils A, B, C, D, E, F, G, and No. 5. At *O* in the figure is shown an *electric oscillator* having a " VT 1 " vacuum tube, a 44-volt B-Battery, interchangeable three terminal coils at *M* used in a Hartley circuit, a gear-driven air condenser and vernier condenser for tuning, with the control handles (not shown) of these condensers projecting through the metal shield in the figure. The electric oscillator *O* had a milammeter (total scale 5 milamperes) in circuit with the grid. This milammeter

was read by a large fixed lens, or reading glass, and was used to show reaction when the wavemeter *WM* was brought into resonance with the oscillator *O*. At *S* is shown the *piezoelectric oscillator* of Figure 1, used with a two-stage audio amplifier and loud speaker. The primary coil of the input transformer of the audio amplifier is connected to the terminals marked "To Phones" in Figure 1. This permits the use of a loud speaker (Magnavox or Western Electric) so that head receivers with their uncertain effects on frequency need not be worn

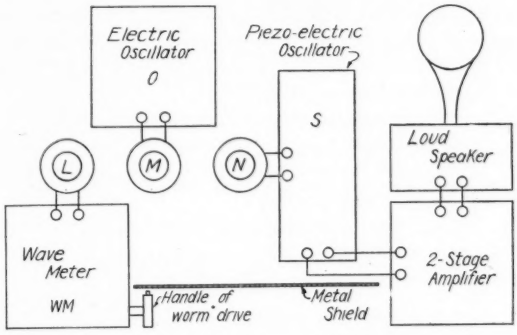

FIGURE 3. Arrangement of Apparatus for Calibration of Wavemeter.

by the observer. The coil *N* has an inductance of 20 or 125 millhenries (either could be used without any change of frequency of the piezoelectric oscillator *S*), and was connected as "Load" in Fig. 1, to replace the 30,000 ohms of resistance.

The procedure of calibration was as follows: the electric oscillator *O* was adjusted in frequency by varying its condenser, beats were obtained between the frequency of *O*, or one of its harmonics, with the frequency of *S*, or one of its harmonics, and the adjustment was brought to the beat zero.

The wavemeter at *WM* having been stationed far enough away from *M* to give only a slight reaction was open circuited and left in position during the preceding adjustment of *O*. Its circuit was then closed and adjusted to resonance with *O*. The resonance was indicated by a small decrease of deflection of the grid milammeter of *O*, or by the absence of a beat note in the loud speaker, whereas for a slight departure from resonance the reaction of the wavemeter upon the coil *M* caused a note. After proper arrangement of the metal shield and

control handles so as to keep the observer at a distance from the electric oscillator O, readings consistent to about one tenth of one per cent could be obtained.

Identification of Inter-harmonic Resonant Frequencies.—Beats occurred when any harmonic of the electric oscillator O (up to the 70th harmonic) was nearly of the same frequency as any harmonic of the piezoelectric oscillator S (up to the 29th harmonic), and a beat zero was obtained when the frequencies of these two harmonics (or fundamentals) were the same.

To identify these harmonics note was taken of the intensities of the beats, and such values as n=1, 2, $\frac{1}{2}$, $\frac{1}{3}$ etc., where n=ratio of wave-length of O to wave length of S, were easily identified by their great intensity and their sequence.

The more complicated resonance relationships, such as $n = 8/29$, indicating that the 8th harmonic of O was resonant with the 29th harmonic of S, could be identified by the use of the interpolation formula

$$\lambda^2 = aC^\circ + b, \tag{1}$$

or

$$n^2 = \alpha C^\circ + \beta, \tag{2}$$

where

λ = wave length of oscillator O,
C° = divisions of wavemeter condenser,
n = ratio of wave length of oscillator O to wave length of oscillator S,
a,b, or α, β = constants determined by two simple values such as values of C° for $n = 1/6$ and $n = 1/3$.

This formula (1) or (2) holds only for the parts of the condenser scale where the capacity calibration against divisions is linear. Outside of this range values for different wavemeter coils were intercompared.

Further facts regarding the interpolation formula are treated later in this report.

Example: Calibration of Wavemeter with Coil B and C.—As an example of this operation, data for coil "B" and "C" are given in Tables I and II. The first column contains settings of the wavemeter condenser (in divisions) corresponding to various adjustments of the oscillator O which gave beat zero with the oscillator S. The second column contains the values of n finally arrived at. The determination of these values of n for C° greater than 3.5 was effected by using formulas of the type of (2) which, with constants determined from easily recognizable values of n, are

TABLE I.

Coil B.

C° Wavemeter Condenser Divisions	n	n^2 observed	n^2 calculated by (3b)	Percentage Departure
3.530	3/11	.07437	.07433	−.06
3.863	2/7	.08163	.08170	+.12
4.135	8/27	.08779	.08790	+.14
4.230	3/10	.09000	.09004	+.03
4.440	4/13	.09467	.09480	+.14
4.783	8/25	.1024	.1024	.00
5.173	1/3	.1111	.1112	+.10
5.603	8/23	.1210	.1209	−.08
6.122	4/11	.1322	.1325	+.20
6.685	8/21	.1451	.1451	.00
7.356	2/5	.1600	.1602	+.10
7.948	5/12	.1736	.1735	−.06
8.117	8/19	.1773	.1773	.00
8.405	3/7	.1837	.1837	.00
9.023	4/9	.1975	.1976	+.05
9.442	5/11	.2066	.2070	+.20
10.090	8/17	.2215	.2216	+.05
11.361	1/2	.2500	.2501	+.04
12.886	8/15	.2844	.2843	−.03
13.450	6/11	.2975	.2970	−.16
13.936	5/9	.3086	.3079	−.23
14.769	4/7	.3265	.3266	+.03
15.362	7/12	.3403	.3399	−.13
16.240	3/5	.3600	.3596	−.11
17.052	8/13	.3787	.3778	−.24
18.240	7/11	.4049	.4045	−.10
20.017	2/3	.4444	.4444	.00
22.052	7/10	.4900	.4900	.00
23.811	8/11	.5289	.5295	+.12

TABLE II.

Coil C.

C° Wavemeter Condenser Divisions	n	n^2 observed	n^2 calculated by (3c)	Percentage Departure
.638	4/13	.09467		
.979	8/25	.1024		
1.158	1/3	.1111		
1.335	8/23	.1210		
1.480	4/11	.1322		
1.651	8/21	.1451	These values of n were identified by intercomparison with Coil B.	
1.841	2/5	.1600		
1.945	7/17	.1695		
2.059	8/19	.1773		
2.304	4/9	.1975		
2.571	8/17	.2215		
2.896	1/2	.2500		
3.268	8/15	.2844		
3.754	4/7	.3265	.3263	.06
4.130	3/5	.3600	.3604	.1
4.322	8/13	.3787	.3778	.3
5.055	2/3	.4444	.4441	.07
5.800	5/7	.5120	.5115	.1
6.007	8/11	.5289	.5303	.25
6.368	3/4	.5625	.5625	.00
7.211	4/5	.6400	.6394	.1
8.872	8/9	.7901	.7898	.04
11.195	1	1.000	1.0001	.01
15.18	7/6	1.362	1.362	.00
16.050	6/5	1.440	1.440	.00
17.39	5/4	1.563	1.561	.13
19.78	4/3	1.778	1.778	.00
21.830	7/5	1.960	1.962	.1
22.675	10/7	2.041	2.040	.05

$$n^2 = .022444C^\circ - .0049, \quad \text{and} \quad (3b)$$
$$n^2 = .090551C^\circ - .0136, \quad (3c)$$

for coils " B " and " C," respectively.

Values of n^2 calculated from these formulas are recorded in column four, and the simple common fractions with squares nearest to them are inserted in column two. It is seen that with the exception of the values at two or three points the two values of n^2 check within two tenths of one per cent. Since n^2 is proportional to λ^2, this identifies λ to 1/10 of one per cent in its relation to the fundamental wave length of the crystal oscillator.

In case of coil " C " values of n for which C° is less than 3.5 were identified comparison with results obtained by the use of the smaller wavemeter coil " B," with which they came within the range of condenser settings for which the linear formula (3b) held.

Similar observations were made with coils " A," " D," " E," " F," " G " of the wavemeter. Finally by the use of coil " No. 5 " ($\frac{1}{2}$ henry) the calibration was extended into the range of audible frequencies, so that the absolute value of the fundamental frequency of the piezoelectric oscillator could be obtained by comparison with a vacuum-tube driven tuning fork.

Concerning Complete Range.— If

λ_0 = electric wave length corresponding to the fundamental mechanical frequency of the piezoelectric oscillator S, and

λ = any wave length of the oscillator O which gives a beat zero, then

$\frac{\lambda}{\lambda_0} = n = \frac{p}{q}$, say, where p and q are integral numbers. This means that the pth harmonic of the electric oscillator O is of the same frequency as the qth harmonic of the piezoelectric oscillator S.

In Table III, some of the combinations of p and q that were actually employed are indicated by an " x "; for example, an " x " at the intersection of the row for $p = 9$ with the column for $q = 11$ indicates that a wave length $\lambda = \frac{9}{11}\lambda_0$ was a calibration point. It is seen that the table extends from $\lambda = \frac{\lambda_0}{13}$ to $\lambda = 70\,\lambda_0$ and this will be seen to be

TABLE III.

Showing What Wavelength Ratios Were Employed.

		q												
		1	2	3	4	5	6	7	8	9	10	11	12	13
p	1	x	x	x	x	x	x	x	x	x	x	x	x	x
	2	x	x	x	x	x	x	x	x	x	x	x	x	x
	3	x	x	x	x	x	x	x	x	x	x	x	x	x
	4	x	x	x	x	x	x	x	x	x	x		x	
	5	x	x	x	x	x		x		x	x	x	x	
	6	x	x	x	x	x	x		x	x	x	x	x	
	7	x	x	x	x	x	x	x			x	x	x	
	8	x	x		x	x	x		x		x	x	x	to $q-29$
	9	x	x	x	x	x	x			x			x	
	10	x	x	x	x	x	x	x	x		x		x	
	11	x	x	x		x		x				x		
	12	x	x	x	x		x		x	x	x		x	
	13	x	x				x		x					x
	14	x	x		x		x	x	x		x		x	
	15	x	x	x		x	x	x		x	x		x	
	16	x												
	17	x												
	18	x												
	19	x												
	.	x												
	.													
	70	x												

from 50 meters to 50430 meters as soon as λ_0 is obtained in the next section.

The range can very easily be extended particularly in the direction

of the shorter waves, and this will be done as soon as the proper wavemeter coils are constructed.

The oscillators for this work have been made up in a permanent form so that these wavemeters may at any time be recalibrated by a series of observations requiring about one-half hour for each wavemeter coil and condenser.

Determination of Absolute Value of Frequency of Piezoelectric Oscillator.— A vacuum-tube drive tuning fork was investigated for me by Professor F. A. Saunders by the use of a phonodyke and found to have a frequency F = 996.3 cycles per second at 21°C. Professor E. L. Chaffee and I measured the same fork by driving an iron gear wheel containing 40 teeth near the poles of a telephone receiver and controlling the speed of the driving motor by a Leeds and Northrup tuning-fork control-apparatus, adjusted so that the frequency of current generated by the toothed wheel synchronized with the vacuum-tube driven fork. The motor speed was then chronographed 3000 seconds. By this we obtained $F = 995.9$.

The average of our value and Professor Saunders's value is

$$F = 996.1\ (\pm .2) \text{ cycles per sec.}$$

The vacuum-tube driven fork here employed differs from that of Professor Eccles in that a microphone button was used to modulate the current to the grid circuit. The advantage of the microphone control is its simplicity of construction and the absence of the possibility of self-excitation by induction independent of the fork. This microphone button was of the inertia-back type and was mounted on the heavy block to which the base of the tuning fork was attached, so that changes of the distribution of mass in the microphone did not affect the period of the fork.

The essentials of the circuit employed with the vacuum-tube driven fork are shown in Figure 4. The apparatus was made by an easy modification of a General Radio 1000-cycle microphone-driven fork. The microphone button was taken from the prong of the fork where it occurs in the General Radio apparatus, and was mounted at its base, marked " Mic." in Figure 4. A 4-volt battery was connected in series with the microphone button and the low side of a modulation transformer. The high side of this transformer was connected to the grid and filament of the vacuum tube, of which the plate circuit contained the B-Battery and driving coil (wound to a high resistance). A condenser of about .2 microfarad was connected in shunt to the driving coil.

In the work of the absolute calibration it was found convenient to dispense with the tube whose elements are shown at *P*, *G*, and *F*,

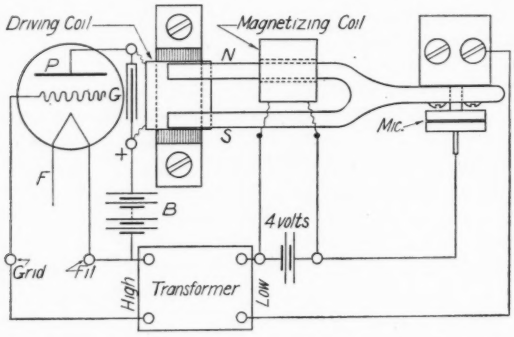

FIGURE 4. Circuits of Vacuum-tube Driven Tuning Fork.

Figure 4, and drive the fork with the two-stage amplifier needed for other purposes with the apparatus, as is shown in Figure 5.

The high-side of the modulation transformer of the tuning fork (see Fig. 4) was connected in series with the plate circuit of the electric

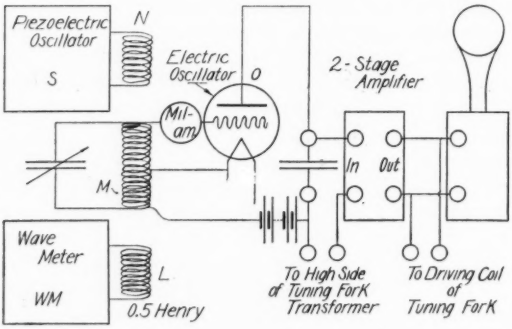

FIGURE 5. Arrangement of Apparatus for Absolute Standardization of Piezoelectric Oscillator.

oscillator *O* and the input terminals (a transformer primary) of the amplifier (see Fig. 5). The terminals of the driving coil of the tuning

fork (Fig. 4) were connected to the output terminals of the amplifier in parallel with the coil of the loud speaker. With this arrangement the amplifier amplified the output of oscillator *O* and also maintained the vibration of the tuning fork.

In Figure 5 approximate positions of the wavemeter *WM* and the piezoelectric oscillator *S*, relative to the electric oscillator *O* are shown. The coil *L* of the wavemeter in this case was $\frac{1}{2}$ henry; the condenser of the wavemeter was the General Radio Precision Air Condenser used in the work described above. The purpose of the oscillator *O* and of the wavemeter in Figure 5 is to permit interpolations between the various multiples of the frequency of the tuning fork and the various fractions of the frequency of the Piezoelectric Oscillator *S*. The procedure was as follows:

The approximate wavemeter setting for the 6th harmonic of the tuning fork (called 6F) was identified, and the oscillator *O* was adjusted so that its fundamental gave a beat zero with this 6th harmonic. The wavemeter *WM* was then resonated to the oscillator by giving its condenser the value 21.817 divisions. We then proceeded with other harmonics of the tuning fork obtaining Table IV.

Following the interpolation law of equation (1) we find that the data of Table IV satisfy with an accuracy of $\frac{1}{10}$ of one per cent the equation

TABLE IV.

Wavemeter (Coil No. 5) in Terms of Tuning Fork.

m F Frequency	C° Wavemeter Condenser Divisions	*m* observed	*m* calculated
6 F	21.817	6	6.000
7 F	15.529	7	7.003
8 F	11.477	8	8.001
9 F	8.693	9	8.999
10 F	6.715	10	9.990
11 F	5.232	11	10.99
12 F	4.104	12	12.00
13 F	3.216	13	13.01

$$\frac{1}{m^2} = .0011758\, C^\circ + .002126, \tag{4}$$

where C° = divisions of wavemeter condenser,
m = coef. of F in Column I (Table IV),
F = frequency of the tuning fork,

as may be seen by a comparison of the third and fourth columns of Table IV.

The next operation consisted in obtaining beat zeros between harmonics of the Electric Oscillator O and the fundamental of the Piezoelectric Oscillator S in the range of frequencies covered by Table IV. This gave the results recorded in Table V.

Here $f_0/70$ means that the 70th harmonic of the electric oscillator gave a beat zero with the fundamental frequency f_0 of the piezoelectric oscillator.

In this table the values of k in the first and third columns were identified by counting beat zeros all the way up from $k=1$. Their identity was established by two independent runs. The second column contains observed values of C° the resonant settings of the wavemeter condenser in divisions, when tuned to the electric oscillator O.

The observations satisfy the empirical interpolation equation

$$k^2 = 208.7\, C^\circ + 377.7, \tag{5}$$

as is seen by a comparison of the third and fourth columns. It is to be noted that an error of 0.2% in k^2 makes an error of only 0.1% in frequency.

If now we divide (5) by f_0^2 and divide (4) by F^2 we have

$$\frac{k^2}{f_0^2} = \frac{208.7 C^\circ}{f_0^2} + \frac{377.7}{f_0^2}, \tag{6}$$

and

$$\frac{1}{m^2 F^2} = \frac{.0011758 C^\circ}{F^2} + \frac{.002126}{F_0^2}. \tag{7}$$

The left hand side of both of these equations is the square of the reciprocal of the frequency corresponding to a given value of C°; hence, equating coefficients, we have

$$\frac{208.7}{f_0^2} = \frac{.0011758}{F^2} \tag{8}$$

TABLE V.

Frequency of Wavemeter (Coil No. 5) in Terms of Fundamental Frequency of Piezoelectric Oscillator.

f_0/k Frequency	C° Wavemeter Condenser Divisions	k^2 obs	k^2 calc	f_0/k Frequency	C° Wavemeter Condenser Divisions	k^2 obs	k^2 calc
$f_0/70$	21.688	4900	4904	$f_0/50$	10.152	2500	2496
69	21.022	4761	4765	49	9.694	2401	2401
68	20.325	4624	4620	48	9.227	2304	2303
67	19.633	4489	4475	47	8.760	2209	2206
66	19.066	4356	4357	46	8.333	2116	2117
65	18.397	4225	4217	45	7.881	2025	2022
64	17.803?	4096	4093	44	7.462	1936	1935
63	17.191	3969	3965	43	7.047	1849	1848
62	16.608	3844	3844	42	6.598	1764	1755
61	16.013	3721	3720	41	6.257	1681	1684
60	15.420	3600	3596	40	5.849	1600	1598
59	14.852	3481	3477	39	5.501	1521	1526
58	14.298	3364	3362	38	5.113	1444	1445
57	13.68?	3249	3233	37	4.753	1369	1370
56	13.194	3136	3131	36	4.388	1296	1294
55	12.665	3025	3021	35	4.071	1225	1227
54	12.131	2916	2909	34	3.725	1156	1155
53	11.650	2809	2809	33	3.391	1089	1085
52	11.141	2704	2703	32	3.084	1024	1021
51	10.640	2601	2598	31	2.795	961	961

and
$$\frac{377.7}{f_0^2} = \frac{.002126}{F^2}. \tag{9}$$

Equation (8) gives $f_0 = 421.28\ F.$ (10)

Equation (9) gives $f_0 = 421.50\ F.$ (11)

Giving greater weight to (10) we may consider

$$f_0 = 421.3\ F$$
$$= 421.3 \times 996.1$$
$$= 419650 \text{ cycles per second;} \qquad (12)$$

whence $$\lambda_0 = 714.9 \text{ meters.} \qquad (13)$$

In these equations f_0 is the frequency of mechanical vibration of the piezoelectric oscillator and λ_0 the corresponding wave length in free space of the electric wave of frequency f_0.

IV. The Linear Interpolation Formula.

Expression of Results of Calibration in Terms of Condenser Divisions.— Having now obtained the absolute value of the frequency f_0 of the piezoelectric oscillator and of the corresponding wavelength λ_0, and having found that an interpolation formula of the type of equation (2) expresses the results with an accuracy of one tenth of one per cent for values of condenser for which its capacity is linear in terms of division, C°, we may change equations such as (3b) and (3c) and (5) into equations for λ^2 by multiplying both sides of these equations by $\lambda^2_0 = (714.9 \text{ meters})^2 = (.7149 \text{ km.})^2$.

This gives within the limits prescribed, the following results for the several wavemeter coils (Table VI).

Table VI.

Equations Connecting Wavelength λ in Kilometers with Condenser Divisions C° Between C° = 3.50 and C° = 23.00.

Coil	Equation
A	$\lambda^2 = .0024778\ C° - .0005724$
B	$\lambda^2 = .011471\ C° - .002504$
C	$\lambda^2 = .046279\ C° - .006951$
D	$\lambda^2 = .2181\ C° - .030$
E	$\lambda^2 = 1.1084\ C° - .024$
F	$\lambda^2 = 4.4597\ C° + .2555$
G	$\lambda^2 = 9.5868\ C°$
No. 5	$\lambda^2 = 106.66\ C° + 193.35$

For values of C° outside of the specified range at the heading of Table VI, values were obtained also by intercomparison of different coils, and the whole result has been compiled in tables not here given.

Expression of Results of Calibration in Terms of Condenser Capacity.— Within the range of wavemeter condenser No. 14 between

3.50 and 23.00 divisions its capacity C in micromicrofarads in terms of divisions C° to better than .2% is given by the linear equation

$$C = 59.60C^\circ - 19.60, \qquad (14)$$

so that the equations of Table VI, with C° in divisions changed to equivalent values in terms of C (in micromicrofarads) take the values of Table VII.

TABLE VII.

Equations Connecting Wavelength λ in Kilometers with C in Micromicrofarads Between C = 189 and C = 1455.

Coil	Equation
A	$\lambda^2 = .00004157\ C + .0002422$
B	$\lambda^2 = .0001925\ C + .001267$
C	$\lambda^2 = .0007765\ C + .00826$
D	$\lambda^2 = .003659\ C + .0417$
E	$\lambda^2 = .01860\ C + .3404$
F	$\lambda^2 = .07483\ C + 1.722$
G	$\lambda^2 = .16085\ C + 3.152$
No. 5	$\lambda^2 = 1.7897\ C + 228.6$

Coefficients of Table VII Examined in Their Relation to Self-Inductance and Natural Wavelength of the Coils.— If the equations of Table VII held for condenser capacity zero the final term of each equation should be λ^2_0 of the coil alone. Also, if the equations held for very large values of C, the coefficients of C should be $4\pi^2 \times 9 \times 10^{-2} L$, where L is the self-induction of each of the coils in henries. These relations give Table VIII.

TABLE VIII.

Regarding Coefficients.

Coil	L Henry Table VII	L measured at 1000 cycles/sec	λ^2_0 natural by Table VII	λ^2_0 natural observed	Tolerance in λ^2_0	Percentage Departure of L
A	.00001170	.000013?	.0002422			
B	.00005417	.000054	.001267			
C	.0002185	.000217	.00826			.70
D	.0010299	.001018	.0417	.0425	.0012	1.18
E	.0052341	.005174	.3404	.3798	.007	1.14
F	.021060	.02079	1.722	1.785	.03	1.28
G	.04527	.04462	3.152	3.338	.07	1.44
No. 5	.5037	.5000	228.6	233.17	1.20	.70

In the next to the last column the item marked "Tolerance in λ^2_0" is the amount by which λ^2_0 may be increased or diminished without affecting the agreement between observed and calculated values of λ within the calibrated range by more than 0.1%. From the fact that the observed and calculated values of λ^2_0 differ by more than the amount of this tolerance it is apparent that the linear law of Table VII does not extend to extremely small values of C.

Regarding L the measured values at 1000 cycles per second for coils "A" and "B" are not accurate beyond the two significant figures given. The percentage by which L calculated by Table VII exceeds the 1000-cycle of L is given in the column marked "Percentage Departure of L."

This Table VIII gives an idea of the error with coils of various sizes and shapes that would arise in the computation of λ from values of L and C measured at 1000 cycles per second and λ_0 measured by direct observations on the free wavelength of the coils.

All of the coils except No. 5 were banked windings designed to minimize distributed capacity. No. 5 was a coil of Maxwellian shape with square cross section closely wound with silk-covered wire.

In practice it is much easier to measure the frequencies and wavelength than the capacities and inductances employed.

Further Note on the Linear Relation of λ^2 to C.— Table IX was obtained with a special single layer coil wound on a glass tube, and with an enormous range of attached capacities. Observed values of λ are compared with values computed from the linear equation

$$\{\lambda_{km}\}^2 = .007349 C_{\mu\mu f} + .0577. \qquad (15)$$

The table shows that with this particular coil this linear equation holds with an accuracy in λ of about 0.1% for values of C ranging from 17 micromicrofarads to 1404 micromicrofarads, and that the range may be extended to an upper value of C equal to 200000 micromicrofarads with an error in λ not greater than about $\frac{1}{2}$ of 1%. This may be largely the error of our measurements of C.

In fact the accurate measurement of C over a wide range presents a difficulty so great that I should recommend the use of the linear relation merely as an interpolation formula to be employed only over a range limited to about 10 to 1 in Capacity.

TABLE IX.

Wavelength for Various Capacities Attached to a Single Layer Coil No. G 3 (with L = .002048 henry).

	n	Resonant value of C in micromicro-farads	λ meters observed	λ meters calculated by (15)	Percentage Departure in λ
Mica Condenser		300400	46430	46990	1.20
		199600	38190	38300	.28
		100800	27030	27220	.70
		20000	12170	12130	.33
G. R. Precision Air Condenser No. 14	9/2	1404	3217	3221	− .12
	4	1105	2860	2860	.00
	7/2	845.6	2502	2504	− .04
	3	618.4	2145	2145	.00
	8/3	487.9	1906	1906	− .15
	5/2	427.3	1787	1788	− .06
	2	271.0	1430	1432	− .14
	7/4	205.4	1251	1252	− .08
	5/3	185.8	1192	1193	− .09
	3/2	149.0	1072.3	1073.6	− .12
	4/3	116.0	953.1	954.0	.10
	5/4	100.7	893.6	893.1	.05
Vernier Condenser	1	61.63	714.9	714.6	.04
	5/6	40.17	595.7	594.1	.26
	3/4	31.23	536.2	536.9	.06
	2/3	23.13	476.6	477.2	− .13
	3/5	17.22	428.9	429.2	− .07
		13.95	395.2	400.3	−1.2
Coil Alone		0.00	202.6	240.4	−20.0

V. Apparatus for Persistent Oscillations of Two Frequencies With a Single Vacuum Tube.

Incidentally it may be noted that the piezoelectric oscillator may be made to give persistent oscillations of two frequencies, with a single vacuum tube, by combining with the crystal a tuned electric circuit of a different frequency. Circuits for attaining this result are shown in Figure 6. This is similar to Figure 5, except that an inductance L_1 and a capacity C_1, in parallel are introduced in series with the piezoelectric crystal, between plate and grid. The inductance L_1 may be

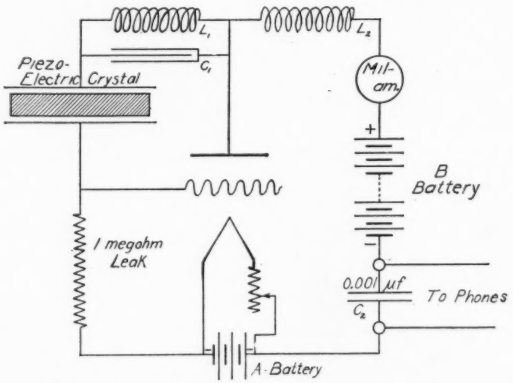

Figure 6. Circuits for Persistent Oscillations of Double Frequency.

in inductive relation to the plate inductance L_2, although in practice I have found that the mutual inductance between L_1 and L_2 may be very small or zero. By adjusting L_1 or C_1 the electric system may be tuned over a wide range of frequencies, in which the system oscillates, while at the same time it is oscillating continuously with a second frequency; namely, the frequency of the piezoelectric crystal.

Beats between those two frequencies, coexisting in the same circuits, occur whenever the fundamental or any harmonic of the electric frequency is made to be nearly equiperiodic with the fundamental or any harmonic of the mechanical period of the crystal; and a beat zero occurs when these frequencies are made exactly equiperiodic.

Thus a single circuit with a single tube may be made to replace the

more elaborate combination of circuits shown in Figure 3, but in the practical calibration of a wavemeter I find Figure 3 somewhat more flexible.

Important applications of Figure 6 are, however, possible. For example, if one wishes to use the beat frequency as a source of audio frequency oscillations, one may cover a large range of audio frequencies with a very small and compact apparatus.

It seems likely that the tuned electrical circuit of Figure 6 may be replaced by one or more piezoelectric crystals, either in series or in parallel with the one shown, so that several frequencies may be produced piezoelectrically with a single vacuum tube. Such a device would meet with wide application.

Rochelle Salt Crystals of very small dimensions (2 x 4 x 10 millimeters) have been successfully employed in the piezoelectric oscillator of Figure 1, to replace the large quartz plate. We are now growing and studying these crystals with respect to their constancy and with respect to their use as sources of multiple frequency with a single vacuum tube.

VI. Piezoelectric Resonators.

We return now to the subject of piezoelectric resonators. As mentioned in the introduction, Professor Cady's paper deals chiefly with this subject. Following his construction the piezoelectric resonator consists of a parallelopiped of quartz one or two square millimeters in cross section and of length dependent on the frequency required.

As to orientation in the crystal, the long dimension [2] of the parallelopiped was, in the natural crystal, oriented perpendicular to two opposite of the six natural faces which hexagonally envelope the crystal. One of the shorter dimensions was in the direction of the optic axis of the crystal (which is perpendicular to the hexagonal base). In this direction the specimen is piezoelectrically *inactive*. The third dimension was perpendicular to the other two dimensions. These three dimensions will be called l, a, and b, respectively.

The parallelopiped is mounted as in Figure 7. Two metal electrodes E are pinned to a hard-rubber base so that they are separated by a space a little wider than the dimension b of the specimen. The crystal specimen is placed between these two electrodes and covered with a hard-rubber cover containing a cavity large enough to leave the crystal

[2] In disregard of strict usage I have here employed the word "dimension" to denote a line parallel to an edge of the parallelopiped.

loose in the cavity. The inactive dimension a is in the direction perpendicular to the static field between the electrodes. A number (e.g. 13.1 in Fig. 7) stamped on each holder designates the approximate length of the several resonators here employed.

As Professor Cady points out, the resonator electrodes may be

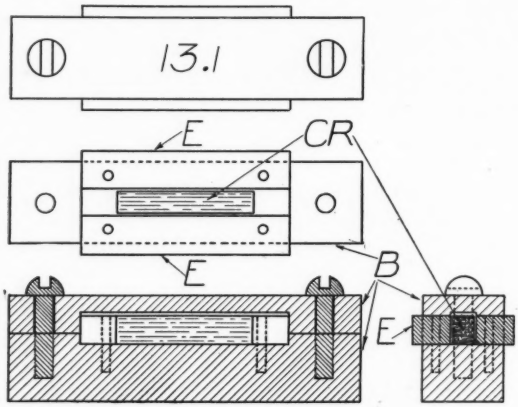

FIGURE 7. Piezoelectric Resonator.

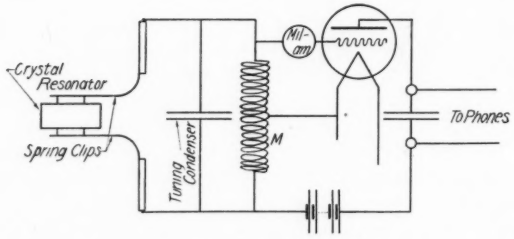

FIGURE 8. Resonator in Clip in Circuit.

brought into contact with various parts of an oscillating electric circuit and will give a response. Figure 8 shows a spring clip within which the electrodes of the resonator may be placed so that the crystal is parallel to the tuning condenser of a Hartley circuit.

With the crystal resonator in place a distinct click or " chirp " is heard on varying the tuning condenser whenever the electrical period of the electrical circuit passes through equiperiodicity with the mechanical period of the crystal resonator. This chirp is a transient phenomenon and by means of it the state of equiperiodicity may be determined with great accuracy.

Table X contains the wavelength λ of the electric wave in free space corresponding to the frequency of the electrical oscillator when it is resonant to the fundamental mode of vibration of several Crystal Resonators of designation given in column I and of actual length given in column II.

TABLE X.

Fundamentals of Crystal Resonators.

Designation No.	Length of Crystal in mm.	meters	meters per mm. length
13.1	13.08	1451	111
12.9	12.90	1429	111
10.82	10.82	1214	112
10.11	10.11	1122	111
8.76	8.67	973.0	112
H 11	8.48	894.9	105
7.62	7.62	856.1	112
7.04	7.04	788.9	112
6.02	6.02	684.3	113
3.1	3.10	332.4	107
		Average	110

In the last column is given the ratio of λ in meters to crystal length in millimeters. This quantity, since determined by the mechanical frequency of the crystal specimen, depends to a slight extent on the width and thickness of the specimen; but is of the order of 110. This gives the velocity of sound in quartz as approximately 5454 meters per second at the high frequencies employed, where no account is

taken of end corrections and corrections for the thickness of the specimen.

In addition to the response corresponding to the fundamental mode of vibration of the crystal resonators, with a node in the middle, they also respond at frequencies corresponding to other modes of vibration having more than one node in the longer dimension, and also to modes of vibration corresponding to the propagation of sound in the direction

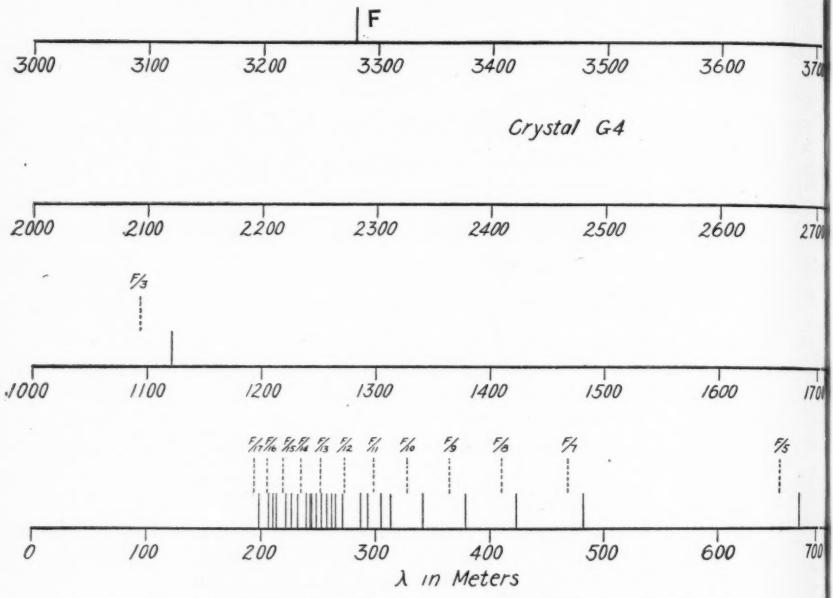

FIGURE 9. Electrical Spectrum of Crystal Resonator G4.

of the static field as well as in the direction of the length. Table XI gives, for each resonator designated No. 13.1, etc., a column of the several wavemeter readings λ in meters of the electric oscillator when resonant to the crystal resonator. The letters S, W, M, indicate whether the response of the crystal is strong, weak or medium, respectively. The table is not complete, as the readings might have been extended much further in the direction of the shorter waves.

The results of a similar set of observations with a crystal receiver, lent me by Professor Cady, and designated " G4 " are plotted as a spectrum in Figure 9.

TABLE XI.

Wavemeter Readings Resonant to Fundamental and Partial Vibrations of Various Crystal Resonators.

No. 13.1	No. 12.90	No. 10.11	H 11	No. 7.04
1450 S	1428.8 S	1121.9 S	893.8	788.9 S
491.2 S	719.8 W	395.8 S	303.6	709.8 W
351.0 S	485.7 S	362.5 W	270.6	697.4 W
346.2 S	358.7 W	348.0 S	219.8	394.7 W
342.9 W	355.9 S	335.8 W	199.3	355.3 W
338.1 W	350.2 W	322.2 W	180.5	350.8 S
334.1 M	283.1 W	255.3 W		347.6 W
305.5 W	270.1 W	222.8 W		333.1 S
284.9 S	258.6 W	212.3 W		327.9 W
281.4 W	256.0 S	203.2 W		311.9 W
270.9 W				276.9 W
269.0 M				262.7 W
265.8 S				260.0 W
263.4 W				258.2 W
				248.9 W
				236.3 S
				230.1 W
				228.3 W
				224.5 W
				219.5 W
				210.9 W
S = Strong				209.5 W
W = Weak				201.3
M = Medium				198.0
				:
				:
				:
				154.8 S
				147.7

The continuous lines at $\lambda = 3280.2, 1120.5$, etc., indicate observed values. For comparison the dotted lines marked F/3, F/5, etc., are values obtained by dividing $F = 3280.3$ by 3, 5, etc., and show where the lines would fall if they were precise submultiples of a single wavelength F. It will be seen that the resonator frequencies are not proportional to the integral members, whereas, on the other hand, the piezoelectric oscillator, as shown in the results above, gives a sustained periodic vibration exactly analyzable into a fundamental frequency and integral multiples thereof.

When a resonator has its lines once tabulated as in Table XI, all the values given may be employed in the subsequent standardization of wavemeters or frequency meters.

CRUFT LABORATORY,
Harvard University, Cambridge, Mass., U. S. A.